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An Analysis for Cobb-Douglas Production Function, the Capability Proxies, Net Society Value and Technology Indicator

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Abstract
This paper establishes an estimate system and forms the capability proxy, the technology capability proxy and the total net society value, etc. and demonstrates the progress of technology can make the capability increase and finds that the technology capability is mainly decided by the ability of the scientists and engineers. The paper completely analyzes the Cobb-Douglas production function and first make out how does it be formed theoretically, what is the k, j, why $k + j$ equals one and demonstrate that in the profit-maximizing condition the excess profit is not necessarily to be zero and there is a bias when we apply of the Cobb-Douglas production function to measure both the capability and the technology.

Keywords: production function, technology, society, profit maximization
JEL Code: O10, O30, O40

I. Introduction

In the economic growth field, several fundamental questions have troubled the researchers until today. The first, it is to be believed that it is the technology make the economic growth but how the technology does the work, it still hasn't been demonstrated theoretically. The second is how to measure the technology, there still hasn't a general technology proxy been found. Some people use the b of the Cobb-Douglas production function as the proxy to measure the technology level but why it can be the proxy? Can it be a good proxy? These also haven't been resolved theoretically. In addition, the b cannot be applied at the company level. The final is what decide the technology capability? Is it decided by the human capital, the total labors, or the machines? It still hasn't been resolved theoretically. To resolve these questions, we should first give an analysis to the Cobb-Douglas production function.

Since the Cobb-Douglas production function was proposed, it has been applied widely and many literatures have been published both in theoretical and the empirical approaches. Although a lot of empirical works meet the function, in the theoretical approach, still some questions haven't been resolved.[1] In the Cobb-Douglas production function

$$p = bL^k \dot{C}^j \qquad (1)$$

The L is the total number of manual workers employed in a manufacturing, the $\dot{C}$ is the fixed capital and the p is the physical production in the manufacturing. What are the k, the j and why the values of the $k + j$ tend roughly to equal unity? As Douglas (1976) mentioned what is the relationship of k, j to a true Labor's share of the product W/P and why they changes, if any, were caused by the business cycle and whether there were long-term changes in k, j, and W/P, as well as alteration in the pattern of residuals.

Can we suppose there would be a still closer degree of agreement between W/P and k (or $k/[k + j]$) if it had been possible to include the payments to all outside workers? Can we think W does not include out-side salesmen, stationary or traveling, nor does it cover carters in the employ of the manufacturing establishments, nor store-men who sell products at retail from the factories, though these men are necessary parts of the production process.

To resolve these questions, make out the economic implication of j and k is critical. It is well known that a prior explanation about the k, j and $k + j$ are: (a) the j and k are the elasticities of production with respect to labor and capital, respectively. (b) If $k + j = 1.0$, the economic system was subject to constant returns to scale. If $k + j$ is greater than 1.0, then a 1 percent increase in both L and $\dot{C}$ would be accompanied by an increase of more than 1 percent in product, and the system as a whole would operate under increasing returns. If $k + j$ is less than 1.0, then the system is characterized by diminishing returns. (c) Marginal physical productivity of labor, for example, declines if $j < 1$ as labor input increase.[2]

This paper, from other point, finds out more economic implications of the j and k and gives more analysis for Cobb-Douglas production function and, furthermore, derives a production function and a series expressions and finally resolves the questions which still has troubled the researchers in the economic growth field.

[1] More details see Douglas, Paul H. (1976), Poirier, Dale J. (1975), Beer, Gerald (1980), Zellner, A. J. Kmenta, and J. Dreze (1966), Simon, Herbert A., and Ferdinand K. Levy (1963), Hoch, Irving (1958), Gunn, Grace T., and Paul H. Douglas (1940), Daly, Patricia, and Paul H. Douglas (1943), Phelps Brown, E. H. (1957), Handsaker, Marjorie L., and Paul H. Douglas (1937), Gunn, Grace T., and Paul H. Douglas (1941) and Samuelson, Paul A. (1979).

[2] More details see Cobb, Charles W., and Paul H. Douglas (1928), Bronfenbrenner, M. and Paul H. Douglas (1939) and Walters, A. A. (1963).

The paper is organized as follows. Section 2 derives out the expression of k, j and $k + j$ and gives an analysis to them in the Cobb-Douglas production function. In Section 3, I derive out a production function. Moreover, I use the function form a capability indicator and the total net society value and analyze the parameters of affecting the R_4 and form a general technology capability indicator. Section 4 provides more discussion. Section 5 offers a conclusion.

II. An Analysis of the k, j and $k + j$ in the Cobb-Douglas Production Function

For easy identification, we replace the p with the $y*$, the total output y minus the total material cost, to express the variant value added, the R replace the b to express the constant, the l replace the L to express the variant number of the labors and set the fixed capital, $\dot{C} = \mathrm{h}\,C$. Where C is the fixed capital cost. According to the Cobb-Douglas production function the K, J also was the constant. Then from equation (1) we get

$$y *= \mathrm{h}^{\mathrm{J}}\mathrm{R}l^{\mathrm{K}}C^{\mathrm{J}} \qquad (2)$$

We take natural logarithms and have

$$log\, y *= log(\mathrm{h}^{\mathrm{J}}\mathrm{R}) + \mathrm{K}\, log\, l + \mathrm{J}\, log\, C$$

and

$$\frac{1}{y*}\frac{\partial y*}{\partial l} = \mathrm{K}\frac{1}{l}$$

$$\frac{1}{y*}\frac{\partial y*}{\partial C} = \mathrm{J}\frac{1}{C}$$

Then we get

$$\mathrm{K} = \frac{l}{y*}\frac{\partial y*}{\partial l} \qquad (3)$$

$$\mathrm{J} = \frac{C}{y*}\frac{\partial y*}{\partial C} \qquad (4)$$

Because the total output

$y = \sum_{i=1}^{n} \mathrm{p}_i = \mathrm{nP}$

The p_i is the price of unit product, thus it can be expressed by the equation

$$\mathrm{p}_i = w_i + a_i + \mathrm{c}_i + b_i + \mathrm{m}_i \qquad (5)$$

Where the i expresses the number of the products; the w_i is the labor cost of producing the ith unit product; the a_i is the marginal profit carried out by the labor for producing the ith unit product; the c_i is the fixed capital cost of producing the ith unit product. Where, we can consider the c_i as the average maintenance cost and average net depreciation of the fixed capital. Therefore, it equals the average fixed capital cost. The b_i is the marginal profit carried out by the capital for producing the ith unit product and the m_i is the marginal material cost of producing the ith unit product. About the a_i and b_i I will discuss them in Section 3.

If the theory of marginal productivity is applicable, and the competition approximately pure, even, a monopolistic circumstance has existed by some companies holding some advanced technologies, before they loss the monopolistic power, the p_i would be a constant. By the iso-cost and iso-quant theory, if in a production period the owner of a firm wants the production profit to be the maximum and the cost the minimum, in the fixed technology condition and the price of other production factors remaining fixed in the market, he may let the among production factors hold a best proportion. It is that in the producing process, the number of labors matches the quantity of fixed capitals and producing materials will approximately hold the best proportion in producing the best amount of products. This means that there must be a minimum production scale unit, in which the number of labors, the quantity of products, fixed capitals and producing materials will approximately hold fixed. If the owner wants to increase or decrease the product quantities, he can increase or decrease the number of production scale units. The Cobb-Douglas production function just meets this condition, because it was formed on the industry level. Therefore, with the number of products increasing, the marginal revenue will remain a constant and the marginal material cost, the average fixed capital cost will remain a constant too.

Therefore
$y = \sum_{i=1}^{n} \mathrm{p}_i = \sum_{i=1}^{n}(w_i + a_i + \mathrm{c}_i + b_i + \mathrm{m_i})$
can be
$y = n\mathrm{p} = \mathrm{n}\overline{w} + n\overline{(a+b)} + n\mathrm{c} + n\mathrm{m}$

Where, c_i and m_i is a constant, so we can set $\mathrm{c}_i = \mathrm{c}$ and $m_i = \mathrm{m}$. p_i is the marginal revenue because it is a constant and equals the price of unit product so

$$\sum_{i=1}^{n} \mathrm{p}_i = n\mathrm{p}$$

In fact, even in the company level, the average cost of the capitals is approximately unchanged if we only consider the machines and some equipments ignoring the land, building and some other fixed capital costs. If we take the land, building and some other fixed capital costs into consideration, based on the microeconomic theory, with the number of the products increasing, the average cost of the fixed capitals will decrease and then increase like the U shape. But in the industry level it still remains constant because the quantities of products increase on the scales. This means in the industry level condition, the fixed capital costs is always proportional to the number of products.

Where, the $\overline{w}$ is the mean value of total labor costs. It equals
$(\sum_{i=1}^{n} w_i)/n$.

The $\overline{(a+b)}$ is the mean value of total profits, that is

$$[\sum_{i=1}^{n}(a_i + b_i)]/n$$

According to the neoclassical marginal productivity theory, the marginal cost of the labor w_i will increase while the a_i will decrease with the number of products increasing until the marginal cost equals the marginal revenue; that is, the a_i plus b_i equals zero. In that condition the total profits will reach the maximum. With the n increases the $\overline{w}$ will increase, the $\overline{(a+b)}$ will decrease but the total profits $n\overline{(a+b)}$ will reach the maximum. Therefore, in this process, both the mean value of total labor costs $\overline{w}$ and the mean value of total profits $\overline{(a+b)}$ are the variant.
Then we have

$y = n\mathrm{p} = n\overline{w} + n\overline{(a+b)} + n\mathrm{c} + n\mathrm{m}$
Thus

$$y - n\mathrm{m} = n\mathrm{p} - n\mathrm{m} = n\overline{w} + n\overline{(a+b)} + n\mathrm{c} \qquad (6)$$

Because the $y - n\mathrm{m}$ is just the value added $y*$, we use $y*$ to replace it and differentiate the equation (6) with respect to n
Gives
$\frac{d(y*)}{dn} = \mathrm{p} - \mathrm{m} = \frac{d(n\overline{w})}{dn} + \frac{d[n\overline{(a+b)}]}{dn} + \mathrm{c}$
Because
$\frac{d(n\overline{w})}{dn} = \frac{dn}{dn}\overline{w} + \frac{d(\overline{w})}{dn}n = \overline{w} + \frac{d(\overline{w})}{dn}n$

$$\frac{d[n\overline{(a+b)}]}{dn} = \frac{dn}{dn}\overline{(a+b)} + \frac{d\overline{(a+b)}}{dn}n = \overline{(a+b)} + \frac{d\overline{(a+b)}}{dn}n$$

Therefore

$$\frac{d(y*)}{dn} = \overline{w} + \frac{d(\overline{w})}{dn}n + \overline{(a+b)} + \frac{d\overline{(a+b)}}{dn}n + \mathrm{c} \qquad (7)$$

From the equation (6) we have

$$\mathrm{p} = \overline{w} + \overline{(a+b)} + \mathrm{c} + \mathrm{m}$$

Then

$$\overline{(a+b)} = \mathrm{p} - (\overline{w} + \mathrm{c} + \mathrm{m})$$

Thus

$$\frac{d\overline{(a+b)}}{dn} = -\frac{d(\overline{w})}{dn} \qquad (8)$$

Substituting the (8) to the (7) we have

$$\frac{d(y*)}{dn} = \overline{w} + \frac{d(\overline{w})}{dn}n + \overline{(a+b)} - \frac{d(\overline{w})}{dn}n + \mathrm{c}$$

Then

$$\frac{d(y*)}{dn} = (\overline{w} + \overline{a}) + (\mathrm{c} + \overline{b})$$

Thus we get

$$d(y*) = (\mathrm{p} - \mathrm{m})dn = (\overline{w} + \overline{a})dn + (\mathrm{c} + \overline{b})dn \qquad (9)$$

In the Cobb-Douglas production function, there are only two parts to compose the increase of the value added $y*$, the increase of the labor part and the increase of the fixed capital part. The increase of the value added in the part of the labor is just the $(\overline{w} + \overline{a})dn$ and in the part of the capital is just the $(\mathrm{c} + \overline{b})dn$.

If we express it in the total differential form, it is the follows

$$dy* = \frac{\partial y*}{\partial l}dl + \frac{\partial y*}{\partial C}dC \qquad (10)$$

Comparing the equation (9) with (10)
We have

$$\frac{\partial y*}{\partial l}dl = (\overline{w} + \overline{a})dn \qquad (11)$$

$$\frac{\partial y*}{\partial C}dC = (\mathrm{c} + \overline{b})dn \qquad (12)$$

Because the number of labors l matches the quantity of fixed capital n will approximately hold the best proportion in the producing process, the l / n must be constant, so we can set it as Q
Then

$$l \,/\, n = \mathrm{Q}$$

$$l = \mathrm{Q}n$$

$$dl = d(\mathrm{Q}n) = \mathrm{Q}dn$$

For the C, the Cobb-Douglas production function formed under the industry level, so the average cost of the capital is approximately unchanged, we can consider it equals the n multiply the average fixed capital cost, the c.
It can be expressed as

$$C = n\mathrm{c}$$

So we have

$$\frac{\partial y*}{\partial n} = \frac{\partial y*}{\partial C} * \frac{\partial C}{\partial n} = \frac{\partial y*}{\partial C} * \frac{\partial (n\mathrm{c})}{\partial n} = \frac{\partial y*}{\partial C}\mathrm{c}$$

Therefore

$$\frac{\partial y*}{\partial n} = \frac{\partial y*}{\partial C}\mathrm{c}$$

Then

$$dC = d(n\mathrm{c}) = \mathrm{c}dn$$

Thus, the equation (11) and (12) became

$$\frac{\partial y*}{\partial l}\mathrm{Q}dn = (\bar{w} + \bar{a})dn \qquad (13)$$

$$\frac{\partial y*}{\partial C}\mathrm{c}dn = (\mathrm{c} + \bar{b})dn \qquad (14)$$

It just meets the chain rule[3]

$$dy* = \frac{\partial y*}{\partial l}\frac{dl}{dn}dn + \frac{\partial y*}{\partial C}\frac{dC}{dn}dn$$

Then we have

$$\frac{\partial y*}{\partial l} = (\bar{w} + \bar{a}) * \frac{1}{\mathrm{Q}} \qquad (15)$$

$$\frac{\partial y*}{\partial C} = \left(\mathrm{c} + \bar{b}\right) * \frac{1}{\mathrm{c}} \qquad (16)$$

Combining the equation (15), (16) with the (3) and (4) respectively we get

$$\mathrm{J} = \frac{C}{y*}\frac{\partial y*}{\partial C} = \frac{n\mathrm{c}}{y*}\left(\mathrm{c} + \bar{b}\right) * \frac{1}{\mathrm{c}} = \frac{n\mathrm{c} + n\bar{b}}{y*} \qquad (17)$$

And

$$\mathrm{K} = \frac{l}{y*}\frac{\partial y}{\partial l} = \frac{l}{y*}(\bar{w} + \bar{a}) * \frac{1}{\mathrm{Q}}$$

Since

$$l = \mathrm{Q}n$$

Then the

$$\mathrm{K} = \frac{\mathrm{Q}n}{y*}(\bar{w} + \bar{a}) * \frac{1}{\mathrm{Q}} = \frac{n(\bar{w}+\bar{a})}{y*} = \frac{n\bar{w} + n\bar{a}}{y*} \qquad (18)$$

Because in the equation (17), the $n\mathrm{c}$ is just the total capital cost, the C, and the $n\bar{b}$ is just the total profit carried by the $n\mathrm{c}$, so we use the B to express the $n\bar{b}$; in equation (18), the $n\bar{w}$ is the total cost of labors and we use the W to express it; the $n\bar{a}$ is the total profit carried by the labors and we express it with the A.
Then we get

$$\mathrm{J} = \frac{C+B}{y*} \qquad (19)$$

$$\mathrm{K} = \frac{W+A}{y*} \qquad (20)$$

But quite often in the producing process, the marginal cost of the labor w_i will also remain constant, we can express it as the w; accordantly the a_i and the b_i remain unchanged too, we use the a and b to express them respectively, in that condition the

$$y = \sum_{i=1}^{n} \mathrm{p}_i = \sum_{i=1}^{n}(w_i + a_i + \mathrm{c}_i + b_i + \mathrm{m}_i)$$

can be

$$y = n\mathrm{p} = n\mathrm{w} + n\mathrm{a} + n\mathrm{b} + n\mathrm{c} + n\mathrm{m}$$

That is

$$y - \mathrm{nm} = y* = n(\mathrm{w} + \mathrm{a}) + n(\mathrm{b} + \mathrm{c})$$

We differentiate it with respect to the labor part, the cost of labors and their carrying profit, that is the l, thinking of the fixed capital part, the cost of capitals and their carrying profit, as a constant
We have

$$\frac{\partial y*}{\partial l} = \frac{\partial[n(\mathrm{w}+\mathrm{a})]}{\partial l}$$

Since

$$n = \frac{1}{\mathrm{Q}}l$$

[3] About the chain rule please see Marsden, Jerrold (1980)

Then

$$\frac{\partial y*}{\partial l} = \frac{\partial y*}{\partial n}\frac{\partial(\frac{1}{Q}l)}{\partial l} = \frac{1}{Q}\frac{\partial y*}{\partial n}$$

So

$$\frac{\partial y*}{\partial l} = \frac{\partial[n(\mathrm{w}+\mathrm{a})]}{\partial l} = \frac{1}{Q}\frac{\partial[n(\mathrm{w}+\mathrm{a})]}{\partial n} = \frac{1}{Q}(\mathrm{w}+\mathrm{a})$$

We get

$$\mathrm{K} = \frac{l}{y*}\frac{\partial y}{\partial l} = \frac{l}{y*}\frac{1}{Q}(\mathrm{w}+\mathrm{a})$$

Then

$$\mathrm{K} = \frac{Qn}{y*}\frac{1}{Q}(\mathrm{w}+\mathrm{a}) = \frac{n\mathrm{w}+n\mathrm{a}}{y*}$$

For the nw and the na being the total cost of labors and the total carrying profit of labors, the K is also the same as equation (20) that is

$$\mathbf{K} = \frac{\boldsymbol{W+A}}{\boldsymbol{y*}}$$

About J, we differentiate the same equation with respect to the fixed capital part, the fixed capital cost and their carrying profit, that is the C, thinking of the labor part, the cost of labors and labors carrying profit, as a constant

We have

$$\frac{\partial y*}{\partial C} = \frac{\partial[n(\mathrm{b}+\mathrm{c})]}{\partial C}$$

Because

$$C = n\mathrm{c}$$

So

$$\frac{\partial y*}{\partial C} = \frac{1}{\mathrm{c}}\frac{\partial y*}{\partial n}$$

and

$$\frac{\partial y*}{\partial C} = \frac{1}{\mathrm{c}}\frac{\partial[n(\mathrm{b}+\mathrm{c})]}{\partial n} = \frac{1}{\mathrm{c}}(\mathrm{b}+\mathrm{c})$$

In the equation (4)

$$\mathrm{J} = \frac{C}{y*}\frac{\partial y*}{\partial C}$$

Substituting $\frac{\partial y*}{\partial C}$ with $\frac{1}{\mathrm{c}}(\mathrm{b}+\mathrm{c})$ to the equation (4) gives

$$\mathrm{J} = \frac{C}{y*}\frac{\partial y*}{\partial C} = \frac{n\mathrm{c}}{y*} * \frac{1}{\mathrm{c}}(\mathrm{b}+\mathrm{c}) = \frac{n\mathrm{c}+n\mathrm{b}}{y*}$$

Because nb is the total profit carried by the nc, it can be expressed by B, and the nc is just C, then we get the **J** as the same as the equation (19) that is

$$\mathbf{J} = \frac{\boldsymbol{C+B}}{\boldsymbol{y*}}$$

Thus, from the expression $\mathrm{K} = \frac{W+A}{y*}$ and $\mathrm{J} = \frac{C+B}{y*}$ we can explain that in the Cobb-Douglas production function $y *= \mathrm{h}^{\mathrm{J}}\mathrm{R}l^{\mathrm{K}}C^{\mathrm{J}}$, the K is the share of the total cost of labors and its carrying profit in the total value added; The J is the share of the total cost of the fixed capitals and its carrying profit in the total value added. This means that in the profit-maximizing condition if $\boldsymbol{B}$ or $\boldsymbol{A}$ is not zero the excess profits is not zero.

We add the J and K then get

$$\mathrm{K}+\mathrm{J} = \frac{W+A}{y*} + \frac{C+B}{y*} = \frac{(W+C)+(A+B)}{y*}$$

From equation (6)

$$y *= y - n\mathrm{m} = n\overline{w} + n\overline{(\mathrm{a}+\mathrm{b})} + nc$$

We have

$$y *= (W+C)+(A+B)$$

Thus we get

$$\mathrm{K} + \mathrm{J} = \frac{(W+C)+(A+B)}{(W+C)+(A+B)} = 1 \qquad (21)$$

This means that the K plus J is the share of the total cost of labors and fixed capitals plus the total profit carried by them in the total value added. Thus the $\mathrm{K} + \mathrm{J}$ must equal one or the J must equal $1 - \mathrm{K}$.

Now, I have answered the question of the relationship of k, j to a true labor's share of the product W/P and demonstrate in the profit-maximizing condition the excess profits is not necessarily zero. As for the R in the Cobb-Douglas production function $y * = \mathrm{h}^{\mathrm{J}}\mathrm{R}l^{\mathrm{K}}C^{\mathrm{J}}$ I will discuss it in the next section of this paper, because it should be combined with the factual production function.

III. Factual Production Function, the Capability Proxy and Technology Indicator, the R in the Cobb-Douglas Production Function

III.I. A Factual Production Function

Actually, in any production process we can have

$$y = \sum_{i=1}^{n} \mathrm{p}_i = \sum_{i=1}^{n} (w_i + a_i + \mathrm{c}_i + b_i + m_i + d_i + o_i) \qquad (22)$$

Then

$$y = W + A + C + B + M + D + O \qquad (23)$$

Where

p_i is the price of unit product.

w_i is the labor cost for producing the ith unit product.

W is the total labor cost.

a_i is the marginal profit carried out by the marginal labor for producing the ith unit product. A is the total profit carried out by the labor.

c_i is the cost of the capital assets in producing the ith unit product. It consists of the cost values of land, buildings, fixtures, machinery, tools, and other equipments. Where, we can consider the c_i as the average net depreciation of the fixed capital and its maintenance cost. Therefore it equals the average fixed capital cost.

C is the total capital cost.

b_i is the marginal profit carried out by the marginal fixed capital for producing the ith unit product.

B is the total profit carried out by the fixed capital.

m_i is the marginal material cost for producing the ith unit product.

M is the total material cost.

d_i is the marginal profit carried out by the marginal material for producing the ith unit product.

D is the total profit carried out by the materials.

o_i is the other marginal cost for producing the ith unit product. It includes the research and development (R&D) cost, the management cost, the advertisement cost and other cost.

From formula (23)

We have

$$y = (W + C + O + M) + (A + B + D)$$

Where, $(W + C + O + M)$ is the total cost and $(A + B + D)$ is the total profit.

Then

$$y = (W + C + O + M)\left[1 + \frac{A+B+D}{(W+C+O+M)}\right]$$

We set

$$\left[1+\frac{A+B+D}{(W+C+O+M)}\right]=\mathrm{R}_1 \qquad (24)$$

Thus, we have the production function

$$y=\mathrm{R}_1(W+C+O+M) \qquad (25)$$

From (23) we also can get

$$y-M=W+A+C+B++D+O$$

Then

$$y-M=(W+C+O)+(A+B+D)$$

We set

$$\left[1+\frac{A+B+D}{(W+C+O)}\right]=\mathrm{R'}_1$$

So we have another production function

$$y-M=\mathrm{R'}_1(W+C+O) \qquad (26)$$

From expression (24) we can see that R_1 will increase while the total profit $(A+B+D)$ increases and the total cost $(W+C+O+M)$ remains unchanged. It means that the more the total profit, the more the R_1. If the total profit $(A+B+D)$ is more than zero the R_1 is more than one. If $(A+B+D)$ equals to zero, then the R_1 equals to one. If the total profit $(A+B+D)$ is less than zero the R_1 is less than one.
From formula (25), we can get another expression of the R_1 :

$$\mathrm{R}_1=y/(W+C+O+M) \qquad (27)$$

Because the $y=n\mathrm{p}$ and the p is the unit product price that is the value of the unit product admitted by the customers, thus y is the total products value admitted by the customers. From expression (27) if the R_1 is less than one, the total products value admitted by the customers is less than the cost of producing it. If the R_1 is more than one, the total products value admitted by the customers is more than the cost of producing it. If R_1 equals to one, the total products value admitted by the customers equals to the cost of producing it. Therefore the R_1 can be the proxy to measure the production capability for a company, an industry and even a nation.

III.II. The R in the Cobb-Douglas Production Function

$$y*=\mathrm{h}^{\mathrm{J}}\mathrm{R}l^{\mathrm{K}}C^{\mathrm{J}} \qquad (2)$$

In the Cobb-Douglas production function condition, that the number of labors matches the quantity of fixed capitals and production materials, will approximately hold the best proportion in producing the best amount of products. Thus, we can set $\frac{C}{l}=\mathrm{t}$. Then we have $C=l\mathrm{t}$.
Also we have $\mathrm{J}=1-\mathrm{K}$. Therefore, the function (2) can be express as

$$y*=\mathrm{h}^{1-\mathrm{K}}\mathrm{R}l^{\mathrm{K}}(l\mathrm{t})^{1-\mathrm{K}}$$

So we have

$$y*=\mathrm{h}^{1-\mathrm{K}}\mathrm{R}l\mathrm{t}^{1-\mathrm{K}} \qquad (28)$$

For the same production, the number of labors matched the quantity of fixed capitals and production materials also holds the same proportion in the function (25), which means

$$C=l\mathrm{t}\,,\; M=l\mathrm{z}$$

is also suitable in the function (25)

$$y = \mathrm{R}_1(W + C + O + M)$$

There are only three parameters, the capital, material and labor, in the production function (2). Thus, the production function (25) can be rewritten as

$$y = \mathrm{R}_1(\overline{w_l} + \mathrm{t} + \mathrm{z})l$$

For the (28), we can get

$$y = \mathrm{h}^{1-\mathrm{K}}\mathrm{R}l\mathrm{t}^{1-\mathrm{K}} + l\mathrm{z}$$

Combining the above two equations we get

$$\mathrm{R} = [\mathrm{R}_1(\overline{w_l} + \mathrm{t} + \mathrm{z}) - \mathrm{z}]/(\mathrm{ht})^{1-\mathrm{K}} \qquad (29)$$

Therefore, the R in the Cobb-Douglas production function (2) has a bias compared with the R_1 if we use it to estimate the technology level. And the R_1 also is not a good technology indicator. We will discuss this in the latter part.

III.III. The Capability Indicator and Total Net Society Value

The expression (27) is most suitable for the investment analysis because it clearly indicates the relationship between the cost and the gain, which investors are most interested in. But, it has a limitation which will cause a bias for us to estimate the capability among the companies, industries and countries if in the unit time, per labor cost is different in different countries, industries and companies, though the personal capability among their labors are the same. Moreover, in some conditions, what the government and the economist care is how many values a company, an industry and a nation can contribute to human being rather than the gain from the cost. If in one working hour, a company has gained more profits than other company only by paying little salary to its labors, we cannot say the capability of this company is higher. So, we have to find a better proxy to deal with this problem.

From formula (23)

$$y = W + A + C + B + M + D + O$$

We have

$$y - (C + M + O) = W + A + B + D$$

Then

$$y - (C + M + O) = W(1 + \frac{A+B+D}{w})$$

We set

$$(1 + \frac{A+B+D}{w}) = \mathrm{R}_2$$

Thus

$$y - (C + M + O) = \mathrm{R}_2 W$$

Because the total labor cost equals the total labor working hours H multiply the average labor cost per hour $\overline{w_h}$; or the total labor number L multiply the average labor cost $\overline{w_l}$, we have

$$w = L\overline{w_l}\ , w = H\overline{w_h}$$

Thus

$$y - (C + M + O) = \mathrm{R}_2 H\overline{w_h} \qquad (30)$$

or

$$y - (C + M + O) = \mathrm{R}_2 L\overline{w_l}$$

Then we have

$$\mathrm{R}_2\overline{w_l} = [y - (C + M + O)]/L$$

or

$$\mathrm{R}_2\overline{w_h} = [y - (C + M + O)]/H$$

We set

$R_2\overline{w_l} = R_3$
$R_2\overline{w_h} = R_4$
Thus

$$R_3 = [y - (C + M + O)]/L \quad (31)$$

$$R_4 = [y - (C + M + O)]/H \quad (32)$$

The R_4 indicates net society value per hour a company produced. It is the most suitable proxy for us to measure the capability of a company, an industry and a nation, because it avoids the labor cost bias and well demonstrates the society value of a company, an industry and a nation contributing to the society and the people.[4] Therefore the R_4 is called the capability indicator.

Because there are often some part-time labors in a company, if we use the R_3 to measure the capability, certain bias will incur. But when the total working hours are unavailable, we have to use the R_3 instead the R_4.
From formula (30)

$$y - (C + M + O) = R_2 H\overline{w_h}$$

If we set the
$R_2 H\overline{w_h} = V$
Then we have

$$V = y - (C + M + O) \quad (33)$$

The V indicates the total net society value a company produced. It measures the amount of the net society values which a company, an industry, or a nation produced. It is better than the GDP to value the production ability of a country for their society, because it takes the production cost into consideration while excluding the labor cost. So we call V as the net society value indicator.
For some companies, they produce many type products and as for the industry and the country, there are also various products exist in. In that condition, been the R_3 is the net society value produced by a labor, the R_3 of these companies should be

$$R_3 = w_{31}R_{31} + w_{32}R_{32} + \cdots + w_{3k}R_{3k}$$

or

$$R_3 = \sum_{j=1}^{k}(w_{3j}R_{3j})$$

Where the R_{3j} is the capability indicator of the company, industry or country to produce the jth type product. The j indicates the type of the product. If there are k types products then the j should be from 1 to k. The w_{3j} is the weighted factor. It equals the number of the labors that produce jth product divided by the total number of the labors that produce the all types products in the company, industry or country. We express it as

$$w_{3j} = L_j/L$$

Therefore we can calculate the R_3 of these companies or the industry and country as

$$R_3 = \sum_{j=1}^{k}(R_{3j}\, L_j/L)$$

So

$$R_3 = \sum_{j=1}^{k}[y_j - (C_j + M_j + O_j)]/L$$

Thus

$$R_3 = [y - (C + M + O)]/L$$

[4] By research for the high technology level, the salary cost is more and for some different countries the labor cost is different. For example in India, the labor cost is up to 30 percent lower compare to countries like Japan, France, England, Germany, and the US. For more details please see Saranga, Haritha (2009) and (ACMA) 2005.

It is the same with the formula (31). We also have calculated $\mathbf{R_4}$ and the result is the same with the formula (32).

III.IV. The Affecting Factors of the $\mathbf{R_4}$ and a General Technology Capability Indicator

Because

$$\mathrm{R}_4 = [y - (C + M + O)]/H$$

$$y - (C + M + O) = A + B + D + W$$

Therefore

$$\mathrm{R}_4 = (\frac{A+B+D+W}{H})$$

Since

$$y = \sum_{i=1}^{n} \mathrm{p}_i = \sum_{i=1}^{n}(w_i + a_i + \mathrm{c}_i + b_i + m_i + d_i + o_i)$$

Then

$$A + B + D + W = \sum_{i=1}^{n}(a_i + b_i + d_i + w_i) = \sum_{i=1}^{n}[\mathrm{p}_i - (\mathrm{c}_i + m_i + o_i)]$$

We get

$$\mathrm{R}_4 = (\frac{\sum_{i=1}^{n}[\mathrm{p}_i - (\mathrm{c}_i + m_i + o_i)]}{H}) \quad (34)$$

$$\mathrm{R}_4 = (\frac{\sum_{i=1}^{n}(a_i + b_i + d_i + w_i)}{H}) \quad (35)$$

From the equation (34), if the price of unit product increases while the total fixed capital cost, material cost and other cost remain unchanging, the R_4 will increase in the same total labor working hours compared with before. If the price of unit product decreases while the total fixed capital cost, material cost and other cost remain unchanging, the R_4 will decrease within the same total labor working hours. When the companies replace with cheaper material to decrease their product price or improve their products, for example, in manufacture industry producers may use the plastics to replace metals in some parts to reduce the material cost and adopt better machines and equipment's to cut the fixed capital cost, while keeping their product price as the same, their competing companies have to reduce their product price so as to sell out their products. Under this condition, the R_4, the capability of their competing companies will decrease. From the equation (35), we can know, during the producing process, if the total profit increases within the same total labor working hours, the R_4 will increase. Reversely, the R_4 will decrease. But what would make these happen and who would do these things? the a_i, b_i, $d_i$and o_i should be discussed furthermore .

In the ancient times, someone found drying grain and grinding it so that it became very easy to make grain into flour. Thus, in the same total labor working hours, expending the equal strength, he could make more flour than with previously. Other people learned from him and could also make more flour. If the other people could earn more from the more flour which was only produced by this method, we can say that the more profit the other people got was created by the person who had innovated the method, but realized and carried out by the others, because the others didn't contribute more except to apply the method. The more profit is the $\triangle a_i$.[5]

Another person invented a mill to grind the grain and produced many mills to sell. Some people bought these mills and used them grinding more grain than before in the same total labor working hours and with equal strength. Thus for these people, if the more profit generated only by using the mill, we can say that the more profit these people got was created by the inventor, but realized and carried out by the mill in the production process. The more profit is $\triangle b_i$. As the cattle power-driven mill was innovated, the works of

[5] This case involves the field of the growth and ideas but deeply discusses beyond the field of this paper; for more discussion, please see Jones, Charles I. (2005)

grinding grain leaved for the people to do only was put the grain on the mail and then wait for collecting the flour. It can be indicated that in the same total labor working hours and expending less strength but the people could gain more flour. We can confirm that the more profit the people got was created by the inventor and carried out by both the cattle and the cattle mill.

Another person developed a new breed of rice which can yield a half of more crops per year. Some farmers bought the seeds of the new breed rice and cultivated them. At the end of the year, they harvested a half of more crops than before in the same total labor working hours and with equal strength. Then for these farmers, if the more profit only brought by using the new breed rice seeds, we can say that the more profit these farmers got was created by the innovator, but realized and carried out by the materials, the new breed seeds, rather than by these farmers in the production process because these farmers contributed nothing except to use the new materials. The more profit is $\triangle d_i$.

There are also the cases that the profit carried out by both the machines and labors. For example, the assembly line is not only an invention of a system for making things, but also that of a production process on labors, thus the profit carried out by both the machines and labors. We can also find out that the factors create profit case in agriculture. For example, if in a good year, the farmers could get more profit than in an ordinary year, then the increased profit was created by the timely wind and rain and all the favorable climatic weathers.

Now the conditions become a bit different from before. The technology history has shown that even the second Industrial Revolution rested as much on industry-based science as on the more common concept of science-based industry [König (1996)]. In modern times, it is the world of engineering with mechanics, iron-making with metallurgy, farming with soil science, mining with geology, water-power with hydraulics, dye-making with organic chemistry, and medical practice with microbiology and immunology. Without these subjects, it is impossible for the innovation of the computer, the truck, the plane, the automatic lathe and mill, … and the gene medication.

Therefore, the different for the more profit brought out by the new innovations is that it is often created by both the scientists and engineers.[6] In chemistry and pharmacy industry, when a new product has been invented, the more profit is created by the scientist and engineer and carried out by the materials. In the manufacture industry, the more profit often realized and carried out by the machine. When a new production process is developed, the more profit often is realized and carried out by the labor. But in some special industry, for example in the software industry, the more profit is created by the scientists and engineers and carried out by engineers because there the engineers are the labors in this field.

From the above discussion if we exclude the special condition such as earthquake, flood, some other disasters and nature advantages, in the perfectly competitive modern industry, the more profits that the company gained from a new technology are created by scientists who make out the useful knowledge and the engineers who make full use of the useful knowledge to innovate new techniques or improve old ones, rather than by the majority of the labor force. This conclusion can be supported by the research work of Mokyr, Joel (2005). Based on the economic history, he explained "the average "quality" of the majority of the labor force – in terms of their technical training – may thus be less relevant to the development and adoption of the new techniques than is commonly believed." furthermore, he pointed out "the role of the size of the population and its "mean" level of human capital should be questioned." So the level of the R_4 reflects the technology ability of the scientists and engineers for a company, an industry and a nation if we exclude the efficiency affection.

[6] About the economic character of the technology and the relationship between technology and the science in the economic field, please see Mokyr, Joel (2005).

Therefore the formula (32)

$$R_4 = [y - (C + M + O)]/H$$

can work as the general technology indicator to measure the technology level for a company, an industry and a nation. But we should make further development to the numerator part in the (32) to let it better reflect the technology characteristics.

We set it as *TV* (the technology value) equals to sales – material cost – machinery and equipment maintenance cost – machinery and equipment depreciation – lease payment for equipment – amortization expense for patents + lease earn for equipment + earn for sell patents – other cost + advertisement cost and then set the $\frac{TV}{H} = R_t$. Therefore we have the technology level formula

$$R_t = \frac{TV}{H} \qquad (36)$$

The advantage of this adjustment is that it well reflects the technology cost and benefit while ignoring the un-technology effect.

The formula (36) is only a measurement to estimate the technology level, but it is the ability of the scientists and engineers to make the innovation for which determines the high or low level of the technology. Therefore, if it can form a function, the R_t is a function of the technology cost of the company, industry and nation. It can be expressed as

$$R_t = f(TC) \qquad (37)$$

TC is the technology cost of the company, the industry or the nation. For a company, it includes research and development cost, machinery and equipment maintenance cost, machinery and equipment depreciation, lease payment for equipment, amortization expense for patents, salary of the technology employee, training expense for labor operating machine, equipment and other cost, the portion of the tax which the nation has been imposed on the company for education, science and technology development and other things concerning technology purposes and so on.

About what is the specific form of the (37), we should do a lot of empirical works, then may make out of it or even can't, but at least we can know the track between the TC and the R_t along the time for a company, an industry or a nation.

III.V. The Effect of the Advertisement for R_t and the Relationship between R_3 and R

Now let's discuss the effect of the advertisement for R_t. If a company made a successful advertisement, then it can sell more goods. This can increase the amount of the production, but the H, the other cost and benefit will also proportionally increase to the technology value accordingly. Moreover, in the technology value, we have excluded the advertisement cost. Therefore, the R_t will remain the same or change a little.
From the (31)

$$R_3 = [y - (t + z)L]/L$$

We can get

$$R_3 = [y/L - (t + z)]$$

Replace the R_3 in the equation

$$y = h^{1-K} R L t^{1-K} + Lz$$

then

$$R = R_3/(ht)^{1-K} + t^K/h^{1-K} \qquad (38)$$

Comparing with the R_3, using the R as a technology capability proxy will incur a bias because the R is not only decided by the technology capability, it also by the h, t and K. But there are many works use the R as a technology capability proxy. Now we have the R_t as the technology capability proxy to be able to make a better estimation work.

From above discussions we can conclude that the technology progress can upgrade the capability of net society value by increasing the price while remaining the total labor working hours, the cost unchanging or decreasing the cost while remaining the price, the total labor working hours unchanging. And the increase of net society value capability indicates the economic growth.

III.VI. Present Technology Character Indicator

Furthermore we can use TV to form a present technology character. Because of the technology spillover and competition effect, a company and an industry of a country and even a nation cannot keep their present advanced technology for a long time. If they can't develop further from their present technology, they will lose their advantage. So there is a time limit for the present advanced technology. Thus we should first calculate the present technology value (PTV), because it can precisely reflect the present technology condition of a company by combining the period of holding advanced technology.

That is

$$PTV = TV + E_1(TV)/(1+r)^1 + E_2(TV)/(1+r)^2 + \cdots + E_T(TV)/(1+r)^T$$

It can be simply expressed as:

$$PTV = \sum_{i=0}^{T} E_i(TV)/(1+r)^i \qquad (39)$$

i: the number of year.

T: the period of holding advanced technology.

$E_i(TV)$: expected market TV in the ith year.

r: The interest rate.

The reason I use the r to replace weighted average cost of capital as opportunity cost is what here we valued is the technology of the company rather than the company itself, and we use the expected market TV to remedy the risk degree in the stochastic condition.

Because in modern industry, the most profits that the company gained from the technology are created by the scientists and the engineers, we have

$$PTV = \mathrm{R}_c(\text{present technoloy cost})$$

The present technology cost (PTC) includes research and development cost, the machinery and equipment maintenance cost, machinery and equipment depreciation, lease payment for equipment, amortization expense for patents, the salary of the technology employee, the training expense for labor operating machine, equipment and others, the portion of the tax which has been imposed on the company by the nation for education, science and technology development and other concerning technology purposes and so on.

Thus, we form a formula for the present technology character proxy R_c as follow:

$$\mathrm{R}_c = \frac{PTV}{PTC} \qquad (40)$$

The R_c only reflects the period character of technology rather than the technology capability because sometimes, the more cost the less gain; sometimes, the less cost the higher gain. For the former situation, the technology development might meet the bottleneck; for the later, it means that the technology gets a new critical field.

IV. A More Discussion

It is known that the Cobb-Douglas production function (1) comes from the Wicksell function[7]

$$p = L^k \dot{C}^{1-k}$$

It is the Douglass who finds that the total wage (WL) is a constant proportion of the output (Y), that is the

$$WL = \alpha Y$$

So Cobb suggests that the function form should be

$$p = bL^k \dot{C}^{1-k}$$

Thus, the Cobb-Douglas production function is an estimated function but it is supported by a lot of empirical works.[8] It means that the relationship among p, L and $\dot{C}$ is existed corresponding to the Cobb-Douglas production function form for which it cannot be denied. Some people try to demonstrate it in theory but failed.[9] They followed the law:

$$Profit = Total\ Revenue - Total\ Cost$$

To set

(41) $$\pi = n\mathrm{P} - (k\mathrm{I} + l\mathrm{W} + n\mathrm{R})$$

π is the excess profits
P is product price
n is output
k is capital
I is the rate of interest
l is labor
W is the wage rate
n is land
R is the rent

In the maximum profit condition, they think

$$\frac{\partial \pi}{\partial l} = \frac{\partial n}{\partial l}\mathrm{P} - \mathrm{W} = 0$$

Then

$$\frac{\partial (n\mathrm{P})}{\partial l} = \mathrm{W} \qquad (42)$$

They set the total revenue $n\mathrm{P} = y$ then the (42) can be

$$\frac{\partial y}{\partial l} = \mathrm{W} \qquad (43)$$

This means that in the profit-maximizing condition no matter what advantage a company is holding, it cannot get any excess profits. It contradicts with the fact so caused criticism.[10] The others[11] set y as the number of the products in

$$y = bl^k \dot{C}^{1-k}$$

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[7] More details please see Samuelson, Paul A. (1979) and Wicksell (1896).

[8] These works are Cobb, Charles W., and Paul H. Douglas (1928), Handsaker, Marjorie L., and Paul H. Douglas(1937) Bronfenbrenner, M., and Paul H. Douglas(1939), Gunn, Grace T., and Paul H. Douglas (1940),(1941), and Patricia Daly and Paul H. Douglas (1943), etc.

[9] For more details please see Heathfield, David F. (1971)

[10] Please see Samuelson, Paul A. (1979). Moreover, from 1926 to 2000, the average annual rate of return on Treasury bill is 3.9 percent in nominal terms and 0.8 percent in real terms; on government bond is 5.7 percent in nominal terms and 2.7 percent in real terms; on corporate bond is 6.0 percent in nominal terms and 3.0 percent in real terms; on common stocks (S&P 500) is 13.0 percent in nominal terms and 9.7 percent in real terms; on small-firm common stocks is 17.3 percent in nominal terms and 13.8 percent in real terms. It means there are excess profits among companies. For more details please see Brealey, Richard A. (2004).

[11] Now this type mistake still popularly exist in many textbooks for example in藤田康範(2009).

then they have $\pi = \mathrm{p}y - (\dot{C} + l\mathrm{W})$
They think
$\frac{\partial \pi}{\partial l} = \frac{\partial y}{\partial l}\mathrm{P} - \mathrm{W} = 0$
then get $\frac{\partial y}{\partial l} = \mathrm{W}$.
They are wrong. Here I give the demonstration. If w is a constant and p, W too, we cannot use $\frac{\partial \pi}{\partial l} = 0$ to find the maximum point, because in that condition, y will proportionally increase with l.
So $\frac{\partial y}{\partial l} = bkl^{k-1}\dot{C}^{1-k}$
Because $C = l\mathrm{t}$ and $\dot{C} = \mathrm{h}\,C$
Then $\frac{\partial y}{\partial l} = bkl^{k-1}(\mathrm{h}l\mathrm{t})^{1-k} = (\mathrm{ht})^{1-k}bkl^{k-1}l^{1-k} = (\mathrm{ht})^{1-k}bk$
It is a constant and $\frac{\partial \pi}{\partial l}$ also is a constant, so there isn't a maximum point in π. Moreover even there is a maximum point exist, but the value of $\frac{\partial y}{\partial l}$ on the maximum point is a specific value rather then the general value of $\frac{\partial y}{\partial l}$, therefore this type method is wrong.
In addition, I should mentioned is that $\frac{\partial y}{\partial l} = \mathrm{W}$ is the one of equations based it the CES function been formed. Therefore the CES function is wrong.[12]
Paul A. also noted their problem but couldn't demonstrate why it was wrong and thought the Cobb-Douglas production function was wrong and suggested a function

$$Y = aL + bC$$

But this function was the Taylor expansion of the Cobb Douglas function which has been derived by Simon, Herbert A. and Ferdinand K. Levy (1963).

Another one, (Durand, D. (1937)) thought that the formula $p = bL^k\dot{C}^{1-k}$ was changed to $p = bL^k\dot{C}^j$ better, so that it can make the exponent of $\dot{C}$ independently determined instead of treating it as the residual in a homogeneous linear equation. With j independently determined, the production function was no longer constrained to be homogeneous of degree 1, but there are still some problems like the Douglas(1976) states "could instead take such form as the actual figures might dictate. If $k + j = 1.0$, the economic system was subject to constant returns to scale. If $k + j$ was greater than 1.0, then a 1 percent increase in both L and $\dot{C}$ would be accompanied by an increase of more than 1 percent in product, and the system as a whole would operate under increasing returns. If $k + j$ was less than 1.0, then the system was characterized by diminishing returns. We corrected the time studies according to the Durand formula and found, interestingly enough, that the sum of $k + j$ was very close to the previous assumption of unity." But being short of theoretic supporting, he could not draw a conclusion.

By analyzing the value created in the production process and combining the neoclassical theory, I found that the problem was caused by they mistook the wage as a constant. In fact, it is a variant in that they assumed maximum profit condition and I followed the law:

$$\mathrm{p}_i = w_i + a_i + \mathrm{c}_i + b_i + \mathrm{m}_i$$

and further divided the profits into two parts: the profit carried out by the labor and the profit carried out by the capital. Finally I derived

$$\mathrm{J} = \frac{C+B}{y*}$$

$$\mathrm{K} = \frac{W+A}{y*}$$

[12] About the formation of CES function, please see Heathfield, David F. (1971)

$$\mathrm{K} + \mathrm{J} = \frac{(W+C)+(A+B)}{(W+C)+(A+B)} = 1$$

and moreover formed a series expression:

1. The production function

$$y = \mathrm{R}_1(W + C + O + M) \qquad (25)$$

2. R_1: a proxy suitable for the investor to measure the production capability for a company, an industry and even a nation.

$$\mathrm{R}_1 = y/(W + C + O + M) \qquad (27)$$

3. R_4 and R_3: the capability indicator.

$$\mathrm{R}_4 = [y - (C + M + O)]/H \qquad (32)$$

$$\mathrm{R}_3 = [y - (C + M + O)]/L \qquad (31)$$

4. b in the Cobb-Douglas production function

$$p = bL^k\dot{C}^j$$

is R in expression

$$\mathrm{R} = \mathrm{R}_3/(\mathrm{ht})^{1-\mathrm{K}} + \mathrm{t}^{\mathrm{K}}/\mathrm{h}^{1-\mathrm{K}} \qquad (38)$$

And it will incur a bias when we apply it to measure both the capability and the technology.

5. V : the total net society value. It is better than the GDP to value the production ability of a nation for their society

$$\mathrm{V} = y - (C + M + O) \qquad (33)$$

6. R_t: the technology level indicator

$$\mathrm{R}_t = \frac{TV}{H} \qquad (36)$$

7. R_c: the present technology character indicator

$$\mathrm{R}_c = \frac{PTV}{PTC} \qquad (40)$$

The measure of the technology level is fundamental in many economic fields, especially related to the technology and the R&D (research and development).

Many people have used various proxies to estimate the technology in their work. For example, Caselli and Coleman (2006) uses the b of the formula

$$p = bL^k\dot{C}^j$$

to evaluate the technology level of a nation in their word technology frontier research work. Timmer, Marcel P. (2003) uses the level of gross fixed capital stock per hour in a country as a ratio of the USA-level to indicate the technology level in the technology development and rate of return to investment work. Soete, L. (1981) uses the innovative inputs and outputs to value a technology in his general test of technological backwardness and productivity growth work. Fagerberg, Jan (1987) uses the average civil R&D percentage of GDP, patenting activity (less domestic) and average GDP per capita as the technology indicators to analyze the technology levels of some countries.[13] Frensch, Richard, and Vitalijia Gaucaite Wittich (2009) apply the variety of capital goods available for production as a direct measure of the state of technology. Kokko, Ari (1994) utilizes the change of average labor productivity as the indicator to measure the change of the technology level. Liu, Zhiqiang (2008) uses the change of productivity as a proxy to measure the change of the technology level. Saranga, Haritha (2009) makes the royalty and know how expenses as a proportion of operating expenses as a proxy for technology intensity.

But such techniques, though being useful in particular applications, are unlikely to be fruitful in cross-industry and cross-country analysis, since they require very detailed data and special modeling efforts for each specific product. About using b of the Cobb-Douglas production function, it only fits in the statistic condition which means that b cannot be applied at the company level. At the company level, the statistic work should be on the production changing on scale condition. If it doesn't on scale we should adjust it to

[13] About the problem of using these proxies to measure technology level, please see Cohen, Wesley M., and Richard C. Levin (1989)

be on scale and then meet the Cobb-Douglas production function requirement, but this violates the statistic requirement. Because if the production of a company changing on scale, then

$p = bL^k \dot{C}^j$
can be
$np = nbL^k \dot{C}^j$
which is simply the same as
$p = bL^k \dot{C}^j$

And in the function, there are two unknown parameters should be solved, b and k, but we only have one data set, so we can't get a result.[14] But R_t and R_c can make the work easier. We can use R_t as the generalizing technology proxies to estimate a company, an industry even a nation and use R_c to research the technology track according to time. Because the couple proxies exclude the labor costs effect and are measured in terms of the monetary form and rid of the bias existing in the b since there is no statistic problem we can apply them for different companies, industries and countries.

Finally I have to say that the Cobb-Douglas production function actually can be formed as follow:

Form $y - n\mathrm{m} = y* = n(\mathrm{w}+\mathrm{a}) + n(\mathrm{b}+\mathrm{c})$

$n = \frac{1}{\mathrm{Q}} l$ and $C = n\mathrm{c}$

We have $y* = \frac{1}{\mathrm{Q}} l(\mathrm{w}+\mathrm{a}) + n(\mathrm{b}+\mathrm{c})$

And $y* = n(\mathrm{w}+\mathrm{a}) + \frac{C}{\mathrm{c}}(\mathrm{b}+\mathrm{c})$

Then get $y* - n(\mathrm{b}+\mathrm{c}) = \frac{1}{\mathrm{Q}} l(\mathrm{w}+\mathrm{a})$ (44)

And $y* - n(\mathrm{w}+\mathrm{a}) = \frac{C}{\mathrm{c}}(\mathrm{b}+\mathrm{c})$ (45)

From (44) and (45) we have $y*[1 - \frac{n(\mathrm{b}+\mathrm{c})}{n\mathrm{p} - n\mathrm{m}}] = \frac{1}{\mathrm{Q}} l(\mathrm{w}+\mathrm{a})$

And $y*[1 - \frac{n(\mathrm{w}+\mathrm{a})}{n\mathrm{p} - n\mathrm{m}}] = \frac{C}{\mathrm{c}}(\mathrm{b}+\mathrm{c})$

Then $y* = \frac{1}{\mathrm{Q}[1 - \frac{\mathrm{b}+\mathrm{c}}{\mathrm{p}-\mathrm{m}}]} l(\mathrm{w}+\mathrm{a})$ (46)

And $y* = \frac{C}{\mathrm{c}[1 - \frac{\mathrm{w}+\mathrm{a}}{\mathrm{p}-\mathrm{m}}]}(\mathrm{b}+\mathrm{c})$ (47)

For (46) and (47) we can have $y*^k = [\frac{1}{\mathrm{Q}[1 - \frac{\mathrm{b}+\mathrm{c}}{\mathrm{p}-\mathrm{m}}]} l(\mathrm{w}+\mathrm{a})]^k$ (49)

And $y*^{1-k} = [\frac{C}{\mathrm{c}[1 - \frac{\mathrm{w}+\mathrm{a}}{\mathrm{p}-\mathrm{m}}]}(\mathrm{b}+\mathrm{c})]^{1-k}$ (50)

We multiply (49) with (50) get

$y* = [\frac{1}{\mathrm{Q}[1 - \frac{\mathrm{b}+\mathrm{c}}{\mathrm{p}-\mathrm{m}}]} l(\mathrm{w}+\mathrm{a})]^k [\frac{C}{\mathrm{c}[1 - \frac{\mathrm{w}+\mathrm{a}}{\mathrm{p}-\mathrm{m}}]}(\mathrm{b}+\mathrm{c})]^{1-k}$

Then $y* = \{[\frac{\mathrm{w}+\mathrm{a}}{\mathrm{Q}[1 - \frac{\mathrm{b}+\mathrm{c}}{\mathrm{p}-\mathrm{m}}]}]^k [\frac{\mathrm{b}+\mathrm{c}}{\mathrm{c}[1 - \frac{\mathrm{w}+\mathrm{a}}{\mathrm{p}-\mathrm{m}}]}]^{1-k}\} l^k C^{1-k}$

Being $\left[\frac{\mathrm{w}+\mathrm{a}}{\mathrm{Q}[1 - \frac{\mathrm{b}+\mathrm{c}}{\mathrm{p}-\mathrm{m}}]}\right]^k \left[\frac{\mathrm{b}+\mathrm{c}}{\mathrm{c}[1 - \frac{\mathrm{w}+\mathrm{a}}{\mathrm{p}-\mathrm{m}}]}\right]^{1-k}$ is a constant, thus we can set it as constant $\mathrm{h}^{\mathrm{J}}\mathrm{R}$

So get $y* = \mathrm{h}^{\mathrm{J}}\mathrm{R} l^k C^{1-k}$

[14] The interested is that the Cobb-Douglas production function is significant on the industry level which indicates there consist of many different production scales but the b, and the k are near unchanging.

That is why $k + j$ equals one and why the Cobb-Douglas production function is supported by a lot of empirical works, because it is a algebra translation from the factual production function $y - n\mathrm{m} = n(\mathrm{w} + \mathrm{a}) + n(\mathrm{b} + \mathrm{c})$.
From the above deriving we can find the mathematic algebra translation will cause some false results.
If we from $y - n\mathrm{m} = y* = n(\mathrm{w} + \mathrm{a}) + n(\mathrm{b} + \mathrm{c})$
and $n = \frac{1}{\mathrm{Q}} l$ and $C = n\mathrm{c}$
to form $y* = \frac{1}{\mathrm{Q}} l(\mathrm{w} + \mathrm{a}) + \frac{C}{\mathrm{c}}(\mathrm{b} + \mathrm{c})$ (51)
then only from the algebra view to set $y*$ as a constant $\mathrm{c}(\mathrm{w} + \mathrm{a})m$ and consider l and C as the variant to get the isoquant line, we can have

$$l = -\frac{\mathrm{Q}}{\mathrm{c(w+a)}}(\mathrm{b} + \mathrm{c})C + \frac{\mathrm{Q}}{\mathrm{(w+a)}} y*$$

which means the isoquant of the production is the straight line.
But if we from the equation $y* = \mathrm{h^J R} l^k C^{1-k}$ to get the isoquant, we have

$$l = y*^{\frac{1}{k}} (\mathrm{h^J R})^{\frac{-1}{k}} C^{(\frac{k-1}{k})}$$

which means the isoquant of the production is the curve. But actually we don't know what the replacing relationship between l the labor and C the machine is. Therefore, if we only basing each equation, ignoring the specific economic condition and taking a step, to analyze the characters of the equation then to make some economic conclusions, we will cause some false results. This type mistake not only exists in economics field but also in almost every field.

The another I want to interpret is that it is not only increase the number of plants can make the production increase on scale, but even in one plant also can make the production increase on scale because once the number of the labors and the fixed capitals have been fixed, the number of the products being produced per hour also have been fixed. If the production has continued day by day and year by year, then the number of the products, the labor costs and the fixed capital costs also will increase on the multiples of the per hour number of them, that is why in the manufacture industry, the most productions increase on scale and why empirical works of Douglas show the $k + j$ equals one.

V. Conclusion

In the economic growth field, several fundamental questions have troubled the researchers until today. The first, it is to be believed that it is the technology make the economic growth but how the technology does the work, it still hasn't been demonstrated theoretically. The second is how to measure the technology, there still hasn't a general technology proxy been found. Some people use the b of the Cobb-Douglas production function as the proxy to measure the technology level but why it can be the proxy? Can it be a good proxy? These also haven't been resolved theoretically. In addition, the b cannot be applied at the company level. The final is what decide the technology capability. Does it decided by the human capital, the total labors, or the machine? It still hasn't been resolved theoretically. In this paper, I based the law of the price composition and divided the profits as two parts to analyze the Cobb-Douglas production function, finally make out how does it be formed, what is the k, what is the j, why $k + j$ equals one and demonstrate that in the profit-maximizing condition the excess profits is not necessarily to be zero. Furthermore, I derived a production function and based this function developed the capability proxy, the total net society value, the technology level proxy, the present technology character indicator. I also analyzed that a bias will incur when we apply the b of the Cobb-Douglas production function to measure both the capability and the technology. Basing on the production process and technology history, I found that the technology capability was mainly decided by the ability of the scientists and engineers rather than the total labors and got out an expression of $\mathrm{R}_t = \mathrm{f(TC)}$. I also demonstrated the progress of technology can make the capability of net society value increase and which makes the economic growth.

References

ACMA. The Automotive Component Manufacturers Association of India. http:// acmainfo.com/docmgr/Status_of_Auto_Industry/Status_Indian_Auto_Industry.pdf.

Beer, G. (1980). The Cobb-Douglas Production Function. *Mathematics Magazine*, 53(1), 44–48.

Brealey, R. A., & Stewart C. M. (2004). *Principles of Corporate Finance*, 155, Singapore: the McGraw-Hill companies.

Bronfenbrenner, M., & Paul H. D. (1939). Cross-Section Studies in the Cobb-Douglas Function, *The Journal of Political Economy*, 47(6), 761–785.

Caselli, F. & Wilbur, J. C. (2006). The World Technology Frontier. *The American Economic Review*, 96(3), 499–522.

Cobb, C. W., & Paul H. Douglas. (1928). A Theory of Production. *The American Economic Review*, 18(1), 139–165.

Cohen, W. M., & Richard C. Levin. (1989). Empirical Studies of Innovation and Market Structure, *Handbook of Industrial Organization Chapter 18*.

Daly, P. & Paul H. D. (1943). The Production Function for Canadian Manufacturers." *Journal of the American Statistical Association*, 38(222), 178–186.

Douglas, P. H. (1976). The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some New Empirical Values, *The Journal of Political Economy*, 84(5), 903–915.

Durand, D. (1937). Some Thoughts on Marginal Productivity with Special Reference to Professor Douglas, *Journal of Political Economy*, 45(6), 740–58.

Fagerberg, J. (1987). A Technology Gap Approach to Why Growth Rates Differ. *Research Policy*, 16(2-4), 87–99.

Frensch, R. & Vitalijia, G. W. (2009). Product Variety and Technical Change. *Journal of Development Economics*, 88(2), 242–257.

Gunn, G. T., & Paul, H. D. (1940). Further Measurements of Marginal Productivity. *The Quarterly Journal of Economics*, 54(3), 399–428.

Gunn, G. T., & Paul, H. D. (1941). The Production Function for Australian Manufacturing. *The Quarterly Journal of Economics*, 56(1), 108–129.

Handsaker, M. L., & Paul, H. D. (1937). The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria. *The Quarterly Journal of Economics*, 52(1), 1–36.

Heathfield, D. F. (1971). *Production Functions*. London and Basingstoke: the Macmillan Press.

Hoch, I. (1958). Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function, *Econometrica*, 26(4), 566–578.

Jones, C. I. (2005). Growth and Idea, *Handbook of Economic Growth Chapter 16*.

Kokko, A. (1994). Technology, Market Characteristic and Spillovers. *Journal of Development Economics*, 43(2), 279–293.

König, W. (1996). Science-Based Industry or Industry-Based Science? Electrical Engineering in Germany before World War I. *Technology and Culture*, 37(1), 70–101.

Liu, Z. (2008). Foreign Direct Investment and Technology Spillovers: Theory and Evidence. *Journal of Development Economics*, 85(3). 176–193.

Marsden, J. & Alan, W. (1980). *Calculus*. pp. 700–703, Menlo Park: The Benjamin/Cummings Publishing Company, Inc.

Mokyr, J. (2005). Long-Term Economic Growth and the History of Technology, *Handbook of Economic Growth Chapter 17*.

Phelps, B. E. H. (1957). The Meaning of the Fitted Cobb-Douglas Function. *The Quarterly Journal of Economics*, 71(4), 546–560.

Poirier, D. J. (1975). On the Use of Cobb-Douglas Splines. *International Economic Review*, 16(3), 733–744.

Samuelson, P. A. (1979). Paul Douglas's Measurement of Production Functions and Marginal Productivities. *The Journal of Political Economy*, 87(5), 923–939.

Saranga, H. (2009). The Indian Auto Component Industry – Estimation of Operational Efficiency and Its Determinants Using DEA. *European Journal of Operational Research*. 196(2), 707–718.

Simon, H. A., & Ferdinand, K. L. (1963). A Note on the Cobb-Douglas Function. *The Review of Economic Studies*, 30(2), 93–94.

Soete, L. (1981). A General Test of Technological Backwardness and Productivity Growth. *Weltwirtschaftliches Archiv*, 117(4), 639–659.

Timmer, M. P. (2003). Technological Development and Rates of Return to Investment in a Catching-up Economy: the Case of South Korea, *Structural Change and Economic Dynamics*, 14(4), 405–425.

Walters, A. A. (1963). Production and Cost Functions: An Econometric Survey. *Econometrica*, 31(1/2), 1–66.

Wicksell, K. "Eim neues Prinzip der gerechen Besteurung.' Finanz- theoretische Untersuchungen, Jena (1896). Translated as "A New Prin- ciple of Just Taxation." In Classics in the Theory of Public Finance, edited by R. A. Musgrave and A. T. Peacock. London: Macmillan, 1958. 'Marginal Productivity as the Basis of Distribution in Economics." Ekon. Tidskrzft (1900): 305-37. Reproduced in Selected Papers on Economic Theory. London: Allen & Unwin, 1958. - .Lectures on Poltical Economy. Vol. 1. London: Macmillan, 1934. Swedish, 1901; German, 1913; English, 1934. ". The 'Critical Point' in the Law of Decreasing Agricultural Produc-tivity." Ekon. Tidshrift (1916): 285–92. Reproduced in Selected Papers on Economic Theory.

Zellner, A. J. K. & Dreze, J. (1966). Specification and Estimation of Cobb-Douglas Production Function Models. *Econometrica*, 34(4), 784–795.

藤田康範. (2009).ビギナーズ ﾐｸﾛ経済学, 119,ミネルヴァ書房.